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Connected End Equitable Domination In Fuzzy Graphs

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Abstract: In this paper, we introduce the end equitable domination and connected end equitable domination of fuzzy graphs. End equitable domination number, connected end equitable domination number and some other relation are found. The characteristic of end equitable domination number and connected end equitable domination are discussed.
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1. INTRODUCTION

L.A. Zadeh[6] introduced the concepts of fuzzy subset of a set as a way for representing uncertainty. Fuzzy graphs were introduced by Rosenfeld[4], who has described the fuzzy analogue of several graph theoretic concepts like paths, cycles, trees, and connectedness. Somasundaram.A and Somasundaram.S[5] discussed domination in fuzzy graphs. They defined domination using effective edges in fuzzy graphs. Nagoor Gani.A., and Chandrasekaran.V.T., [3] defined domination in fuzzy graphs using strong arcs. The concept of equitable domination in fuzzy graphs was introduced by Dharmalingam and Rani[1]. In this paper, the end equitable domination sets, connected end equitable domination sets and its numbers are defined and discussed

2. PRELIMINARIES

Definition 2.1

A fuzzy graph is defined by $G = (V, \sigma, \mu)$ where V is a vertex set, σ is a fuzzy subset of V and μ is fuzzy relation σ. i.e., $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$.

Definition 2.2

Let $G = (\sigma, \mu)$ be a fuzzy graph then the *order* and *size* are defined as $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{(u,v) \in E} \mu(u, v)$

Definition 2.3

An arc (u, v) in a fuzzy graph $G = (\sigma, \mu)$ is said to be strong if $\mu^{\infty}(u, v) = \mu(u, v)$then u, v are called *strong neighbours*.

Definition 2.4

The *strong neighbourhood* of the vertex u is defined as $N_S(u) = \{v \in V \mid (u, v) is\ a\ strong\ arc\}$.

Definition 2.5

A connected acyclic fuzzy graph is said to be a *tree*.

Definition 2.6

A vertex in a fuzzy graph having only one neighbour is called a *pendent vertex*. Otherwise it is called *non – pendent vertex*.

Definition 2.7[2]

A vertex $u \in V$ dominates $v \in V$ if (u, v) is a strong arc. A subset D of V is called a *dominating set* of a fuzzy graph G if for every $v \in V - D$, there exists $u \in D$ such that u dominates v. The minimum scalar cardinality taken over all dominating set is called *domination number* and it is denoted by γ of a fuzzy graph G.

Definition 2.8[2]

Let u and v be two vertices in a fuzzy graph G. A subset D of V is called an *equitable dominating set* if for every $v \in V - D$ there exist a vertex $u \in D$ such that $uv \in E(G)$ and $|d(u) - d(v)| \leq 1$ and $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$. The minimum scalar cardinality of an *equitable dominating set* in a fuzzy graph is called equitable domination number and is denoted by $\gamma_e(G)$.

3. END EQUITABLE DOMINATION IN FUZZY GRAPHS

Definition 3.1

Let $G = (V, \sigma, \mu)$be a connected fuzzy graph. An equitable dominating set D is said to be an *end equitable dominating set* of G if D contains all the end vertices.

Definition 3.2

The minimum fuzzy cardinality of an end equitable dominating set is called the *end equitable domination number* of G and it's denoted by $\gamma_{ee}(G)$.

Example 3.3

From figure (1) end equitable dominating set $D = \{v_1, v_6, v_3, v_7\}$, $\gamma_{ee}(G) = 2.0$
Minimal end equitable dominating sets $\{v_1, v_6, v_4, v_7\}$ with $\gamma_{ee}(G) = 2.3$ and $\{v_1, v_6, v_8, v_3\}$ with $\gamma_{ee}(G) = 2.1$

Theorem 3.7 *If G is totally equitable disconnected fuzzy graph then $\gamma_e(G) = \gamma_{ee}(G) = p$ for any connected fuzzy G is not isomorphic to K_2.*

Proof Let $G = (V, \sigma, \mu)$be a connected fuzzy graph with order p. Suppose G is not totally equitable disconnected graph then there exist atleast two equitable adjacent vertices x and y have the strong edges then $V - \{x\}$ or $V - \{y\}$ is end equitable dominating set then $\gamma_{ee}(G) \le p - max_{x,y \in V}\{\sigma(x), \sigma(y)\}$ which is a contradiction. Hence G is totally equitable disconnected graph

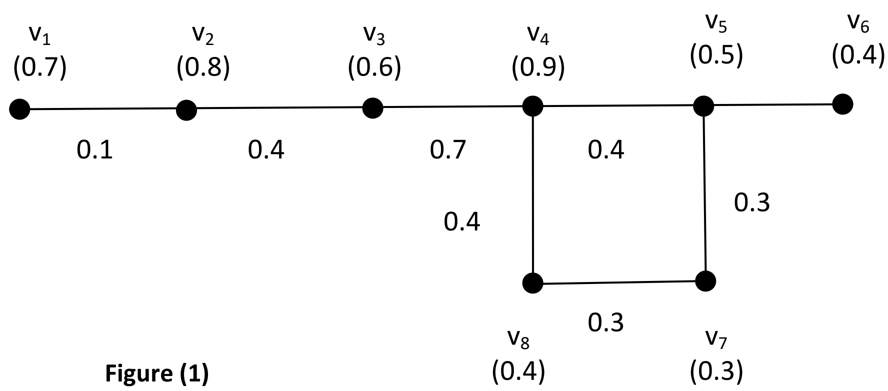

Figure (1)

Theorem 3.4 *For any fuzzy graph $\gamma(G) \le \gamma_e(G) \le \gamma_{ee}(G)$*

Proof Let $G = (V, \sigma, \mu)$be a connected fuzzy graph and $D \subseteq V(G)$ be any minimum end equitable dominating set. By the definition every vertices $x \in D$ has fuzzy end node if it has atmost one strong neigbour in $V - D$ and every vertex the condition $|d(x) - d(y)| \le 1$ for $x \in D$ and $y \in V - D$. Therefore $\gamma_e(G) \le \gamma_{ee}(G)$. Also any equitable dominating set in G is also dominating set. Therefore $\gamma(G) \le \gamma_e(G)$. Hence $\gamma(G) \le \gamma_e(G) \le \gamma_{ee}(G)$.

Theorem 3.5 *Let G be the fuzzy cycle C_n with n vertices, then $\gamma(G) = \gamma_e(G) = \gamma_{ee}(G)$*

Proof Let G be the fuzzy cycle then it is 2-regular fuzzy graph then $\gamma(G) = \gamma_e(G)$ and similarly the fuzzy cycle C_n not have the end vertices. So that $\gamma_e(G) = \gamma_{ee}(G)$. Hence $\gamma(G) = \gamma_e(G) = \gamma_{ee}(G)$.

Observation 3.6

For any fuzzy graph G without pendent vertices then $\gamma_e(G) = \gamma_{ee}(G)$

Remark 3.8 The converse of the above theorem is true but not always. i.e., if $\gamma_e(G) = \gamma_{ee}(G) = p$ then G is totally equitable disconnected.

Theorem 3.9 *For $G = (V, \sigma, \mu)$ be a connected fuzzy graph order p with pendent vertices then $\gamma_{ee}(G) = p - \{\sigma(x)\}$ where x be the pendent vertex if and only if either G has atmost two equitable edges must be adjacent and atleast one of them pendent edge or one equitable edge not pendent.*

Proof Let G be a fuzzy graph with pendent vertices and G has one equitable edge not pendent. Suppose $e = xy$, strong edge then there are only two minimum end equitable dominating sets $V - \{x\}$ or $V - \{y\}$. Therefore $\gamma_{ee}(G) \le p - max_{x,y \in V}\{\sigma(x), \sigma(y)\}$. Similarly if there is one equitable pendent edge $= yx$. $V - \{x\}$ is the only minimum end equitable dominating set of G. Therefore $\gamma_{ee}(G) \le p - \sigma(x)$. If there are two adjacent equitable edges and one of them is pendent say $e = xy$ and the $f = yz$ is not pendent, then $V - \{y\}$ and $V - \{z\}$are the only minimum end equitable dominating sets in G. Thus $\gamma_{ee}(G) = p - \sigma(x)$. Next, consider that the two equitable edges are pendent then $\cong K_{1,2}$. Therefore $\gamma_{ee}(G) = p - \sigma(x)$. Conversely, suppose that G be a graph with

$\gamma(G) = min\,\sigma(x)$ and with order p then $\gamma_{ee}(G) = p - \sigma(x)$. Let $D = V - \{y\}$ be the minimum end equitable dominating set of G. For some vertex y clearly y will not pendent vertex from the definition of the end equitable dominating set. Now the vertex y have two cases following,

Case (1) : y is supporting vertex for some end vertex x, then y must be equitable adjacent by the pendent vertex x and then G has only one pendent equitable edge or y is equitable adjacent by the pendent vertex x and equitable adjacent by another vertex z and hence xy and yz are two equitable adjacent edges.

Case (2) : y is not supporting vertex then from the definition of end equitable domination number since $\gamma_{ee}(G) = p - \sigma(x)$ there is one vertex in D say z must be equitable adjacent to y. Therefore G has only one equitable edge.

Theorem 3.10 *If G be a fuzzy graph then $\gamma_e(G) = \gamma_{ee}(G)$ for any end vertex is equitable isolated vertex.*

Proof Let G be a fuzzy graph and $U \subseteq V(G)$ has set of all only end vertices. Suppose that $D \subseteq V(G)$ is any minimum end equitable dominating set in G. Clearly, every end vertex is also equitable isolated vertex. Assume that $\gamma_e(G) \neq \gamma_{ee}(G)$. That means atleast one end vertex say $y \in F$ is not exist in D. But $d(y) = 0$ then it's contradiction that $y \in D$. Hence any end vertex in G must belong to every minimum equitable dominating set. Therefore any equitable dominating set is end equitable dominating set in G. Hence $\gamma_e(G) = \gamma_{ee}(G)$.

Theorem 3.11 *For $G = (V, \sigma, \mu)$ be fuzzy graph with every vertex adjacent to minimum two vertices then $\gamma_{ee}(G) = p - max_{x_i \in V}\{\sigma(x_i), i = 1,2,..n\}$ where p is the order of G if and only if G has only one equitable strong edge.*

Proof Let G be a fuzzy graph with p is the order of a fuzzy graph then $\gamma_{ee}(G) = p - \sigma(x)$. Next we have to prove that G has only one equitable edge and G has two equitable strong edges have the following cases :

Case (1) : Let us consider the two strong edges are adjacent say $e_1 = xy$ and $e_2 = yz$ then clearly $V - \{x, z\}$ is an end equitable dominating set it means $\gamma_{ee}(G) \leq p - \{\sigma(x) + \sigma(z)\}$ and which is the contradiction. Hence G has only one equitable strong edges.

Case (2) : Now we consider the two equitable strong edges are not adjacent say $e_1 = xy$ and $e_2 = yz$ then it is shows that $V - \{x, z\}$ is an end equitable dominating set of G. which means $\gamma_{ee}(G) \leq p - \{\sigma(x) + \sigma(z)\}$ is a contradiction. Hence there is only one equitable strong edges. Conversely, if G be a fuzzy graph with only one equitable edge and the every vertices in a fuzzy graph G adjacent to minimum two vertices. Suppose $e = xy$ be the only one equitable strong edge then there is only two minimum end equitable dominating set $V - \{x\}$ and $V - \{y\}$ all of the order is $p - max_{x,y \in V}\{\sigma(x), \sigma(y)\}$. Hence $\gamma_{ee}(G) = p - max_{x_i \in V}\{\sigma(x_i)\}$ for $i = 1,2,..n$ be the number of vertices.

Theorem 3.12 *Let Gbe a fuzzy graph then $\gamma(G) = \gamma_e(G) = \gamma_{ee}(G) = min_{x_i \in V}\{\sigma(x_i), i = 1,2,..n\}$ if and only if $G \cong K_n, n \geq 3$ by deleting j independent edges.*

Proof If $G \cong K_n$for any positive integer n, then $\gamma_{ee}(G) = min_{x_i \in V}\{\sigma(x_i), i = 1,2,..n\}$ and then $\gamma(G) = \gamma_e(G) = \gamma_{ee}(G)$. Similarly if G is a graph can be constructed from $K_n, n \geq 3$ by deleting j independent edges from K_n then there exist one vertex of G have adjacent to $(n - 1)$ vertices and the remaining vertices either of adjacent to $(n - 1)$ or of adjacent to $(n - 2)$ vertices. Hence $\gamma_{ee}(G) = min_{x_i \in V}\{\sigma(x_i), i = 1,2,..n\}$. Conversely, let G be a fuzzy graph such that $\gamma(G) = \gamma_e(G) = \gamma_{ee}(G) = min_{x_i \in V}\{\sigma(x_i), i = 1,2,..n\}$. We have the two cases :

Case (1) : G has atleast one vertex adjacent to less than or equal to one and $\gamma_{ee}(G) = min_{x_i \in V}\{\sigma(x_i), i = 1,2,..n\}$ then clearly either $G \cong K_n, n \geq 3$

Case (2) : For any vertex $v \in V(G)$ has been adjacent to greater than or equal to two vertices then $\gamma(G) = \gamma_e(G) = \gamma_{ee}(G) = min_{x_i \in V}\{\sigma(x_i), i = 1,2,..n\}$, there is one vertex must be adjacent to $(n - 1)$ vertices, where n is the number of vertices of G and other $(n - 1)$ vertices have many possibilities as following :

i. All the $(n - 1)$ vertices have the same degree
ii. There are $(n - 3)$ vertices are adjacent to $(n - 1)$ vertices and two vertices are adjacent to $(n - 2)$
iii. Similarly as in (i) and (ii) there are $(n - 5)$ vertices are adjacent to $(n - 1)$ and four vertices are adjacent to $(n - 2)$ and so on.

That means G has some vertices are adjacent to $(n-1)$ and some other of degree $(n-2)$ and atleast one vertex must be adjacent to $(n-1)$ vertices. So it shows that graph G can be constructed from $G \cong K_n, n \geq 3$ by deleting j independent edges from K_n.

4. CONNECTED END EQUITABLE DOMINATION NUMBER

Definition 4.1 Let $G=(V,\sigma,\mu)$ be a fuzzy graph. An end equitable dominating set $D \subseteq V(G)$ is called *connected end equitable dominating set* if the induced subgraph $\langle D \rangle$ is connected.

Definition 4.2 Any connected end equitable dominating set with minimum fuzzy cardinality is called minimum *connected end equitable domination number* and it's denoted by $\gamma_{cee}(G)$

Theorem 4.3 *For any connected fuzzy graph* $G=(V,\sigma,\mu)$ *then* $\gamma(G) \leq \gamma_e(G) \leq \gamma_{ee}(G) \leq \gamma_{cee}(G)$

Proof By theorem, $\gamma(G) \leq \gamma_e(G) \leq \gamma_{ee}(G)$ it is obvious. For any fuzzy graph $D \subseteq V(G)$ be any minimum connected end equitable dominating set in G. clearly D is also an end equitable dominating set in G. Therefore $\gamma_{ee}(G) \leq \gamma_{cee}(G)$. Hence $\gamma(G) \leq \gamma_e(G) \leq \gamma_{ee}(G) \leq \gamma_{cee}(G)$.

Remark 4.4 The above theorem equality condition holds for fuzzy graph has no end vertices. For example, fuzzy cycle and complete fuzzy graph holds the equality condition.

Theorem 4.5 *Let G be a fuzzy graph then* $\gamma_{ce}(G) \leq \gamma_{cee}(G)$

Proof Let D be the minimum connected end equitable dominating set of the connected fuzzy graph G. The dominating set D induced subgraph $\langle D \rangle$ equitable and connected. Therefore D be any connected end equitable dominating set is also connected equitable dominating set. Hence $\gamma_{ce}(G) \leq \gamma_{cee}(G)$

Example 4.6 From figure (2) connected end equitable dominating set $D=\{v_2, v_4, v_3, v_5, v_6\ v_7, v_8\}$, $\gamma_{cee}(G)=4.4$

Theorem 4.7 *For any $r-$ regular fuzzy graph for $r > 1$ then* $\gamma_{ce}(G)=\gamma_{cee}(G)$

Proof Suppose G be a regular fuzzy graph then every vertex of G is same degree say r. Let D be the minimal connected equitable dominating set of G, then $|D|=\gamma_{ce}(G)$. If $x \in V-D$ then D is connected equitable dominating set, then there exist $y \in D$ and xy be the strong edge, also $d(x)=d(y)=r$. Therefore $|d(x)-d(y)|=0 \leq 1$. Hence D is an fuzzy connected end equitable dominating set of G such that $\gamma_{ce}(G) \geq \gamma_{cee}(G)$. Also we have $\gamma_{ce}(G) \leq \gamma_{cee}(G)$. Therefore $\gamma_{ce}(G)=\gamma_{cee}(G)$.

Corollary 4.8 *If G be $(r,r+1)$ bi-regular fuzzy graph then,* $\gamma_{ce}(G)=\gamma_{cee}(G)$

Proof By theorem (4.5)$\gamma_{ce}(G) \leq \gamma_{cee}(G)$, now let D be minimum connected end equitable set of $(r,r+1)$ bi-regular fuzzy graph. By the definition of connected end equitable dominating set D is also equitable dominating set and $\langle D \rangle$ is connected. Since G is $(r,r+1)$ bi-regular fuzzy graph, that means D is also connected end equitable dominating set.

Theorem 4.9 *Let $G=(V,\sigma,\mu)$ be a fuzzy graph then $\gamma_{cee}(G)=\gamma_{ee}(G)=min_{x\in V}\,\sigma(x)$ if and only if G has no end vertex and there is atleast one vertex $v \in V$ adjacent to $(n-1)$ vertices in G, where n is the number of vertices in G.*

Proof For any fuzzy graph $G=(\sigma,\mu)$be connected without end vertices and every vertex adjacent to atleast two vertices and there exist one vertex $v \in V$ in G has adjacent to $(n-1)$ vertices, then the set D is connected end equitable dominating set G we get $\gamma_{cee}(G)=\gamma_{ee}(G)=\min_{x\in V}\sigma(x)$. Conversely, suppose G is connected graph and $\gamma_{cee}(G)=\gamma_{ee}(G)=\min_{x\in V}\sigma(x)$ then G has no end vertex and there is D which is connected end equitable dominating set. Therefore atleast any one vertex adjacent to $(n-1)$ vertices in G.

Theorem 4.10 *If T be any fuzzy tree with order p or G be totally equitable disconnected then $\gamma_{cee}(G)=p$.*

Proof Let $G=(\sigma,\mu)$ be a fuzzy tree, if $D \subseteq V(G)$ then D is connected end equitable dominating set. Therefore, $\gamma_{cee}(G) \leq p$. Now $\gamma_{cee}(G) \leq p-\sigma(x)$ that means any one vertex $x \in V$ doesnot belong to the minimum end equitable dominating set. Then $D \subseteq V(G)-\{x\}$ is connected end equitable dominating set and the vertex $x \in V$ be the adjacent to atleast two vertices then $\langle V(G)-\{x\}\rangle$ is connected then G has only one possible if the induced subgraph $\langle V(G)-\{x\}\rangle$ will be fuzzy cycle. Which is a contradiction to that G is a tree. Because every vertex other then the end vertex in a tree is cut vertex. Hence there is no connected end equitable domination number is less than $p - \sigma(x)$. Hence $\gamma_{cee}(G) = p$. Similarly, if G is equitable totally disconnected then $\gamma_{cee}(G) = p$.

Corollary 4.11 *For any G be a fuzzy cycle with order p then $\gamma_{cee}(G) = p - max_{x,y\in V}\{\sigma(x), \sigma(y)\}$ and $V - \{x, y\}$ should be connected.*

Proof Since $\gamma_{cee}(G) = p - \max_{x,y\in V}\{\sigma(x), \sigma(y)\}$ and $V - \{x, y\}$ is any subset of the vertices on the cycle G such that x and y any adjacent vertices. Clearly D is connected and equitable set of G it means $\gamma_{cee} \leq p - \max_{x,y\in V}\{\sigma(x), \sigma(y)\}$ and the known result we have $p - \max_{x,y\in V}\{\sigma(x), \sigma(y)\} = \gamma_{ce}$. Hence $\gamma_{cee} = \gamma_{cee}(G) = p - \max_{x,y\in V}\{\sigma(x), \sigma(y)\}$.

Corollary 4.12

(i) For any bi-star fuzzy graph $B(p_1, p_2)$ with order $p = p_1 + p_2$ then $\gamma_{cee}(B(p_1, p_2)) = p_1 + p_2$

(ii) For any spider fuzzy graph G then $\gamma_{cee}(G) \leq 2p + 1$

Theorem 4.13 For any complete bipartite fuzzy graph then

$$\gamma_{ce}(G) = \gamma_{cee}(G) = \left\{\begin{matrix} p_1 + p_2, & for\ |m+n| \geq 2 \\ \min_{x\in V}\sigma(x) + \min_{y\in V-\{x\}}\sigma(y), & otherwise \end{matrix}\right\}$$

Where p_1 be the order of the partition with m number of vertices and p_2 be the order of the partition with n number of vertices.

Proof

Case (1) : If $G \cong K_{m,n}$ and $|m + n| \geq 2$, in this case of the fuzzy graph G is equitable totally disconnected and by known result $\gamma_{ce}(G) = p_1 + p_2$ and by theorem ()$\gamma_{ce}(G) \leq \gamma_{cee}(G)$. Hence $\gamma_{ce}(G) = \gamma_{cee}(G) = p_1 + p_2$

Case (2) : If $G \cong K_{m,n}$ where $|m + n| \leq 1$ then if A, B be the partite sets of G, be selecting one vertex $x \in A$ and $y \in B$ then $D = \{u, v\}$ is connected end equitable dominating set. $\gamma_{ce}(G) = \gamma_{cee}(G) \leq \min_{x\in V}\sigma(x) + \min_{y\in V-\{x\}}\sigma(y)$ but $\gamma_{cee}(G) \neq \min_{x\in V}\sigma(x)$. Hence $\gamma_{ce}(G) = \gamma_{cee}(G) = \min_{x\in V}\sigma(x) + \min_{y\in V-\{x\}}\sigma(y)$

Observation 4.14 For any connected fuzzy graph G any minimum connected end equitable dominating set must be contained all the support vertices.

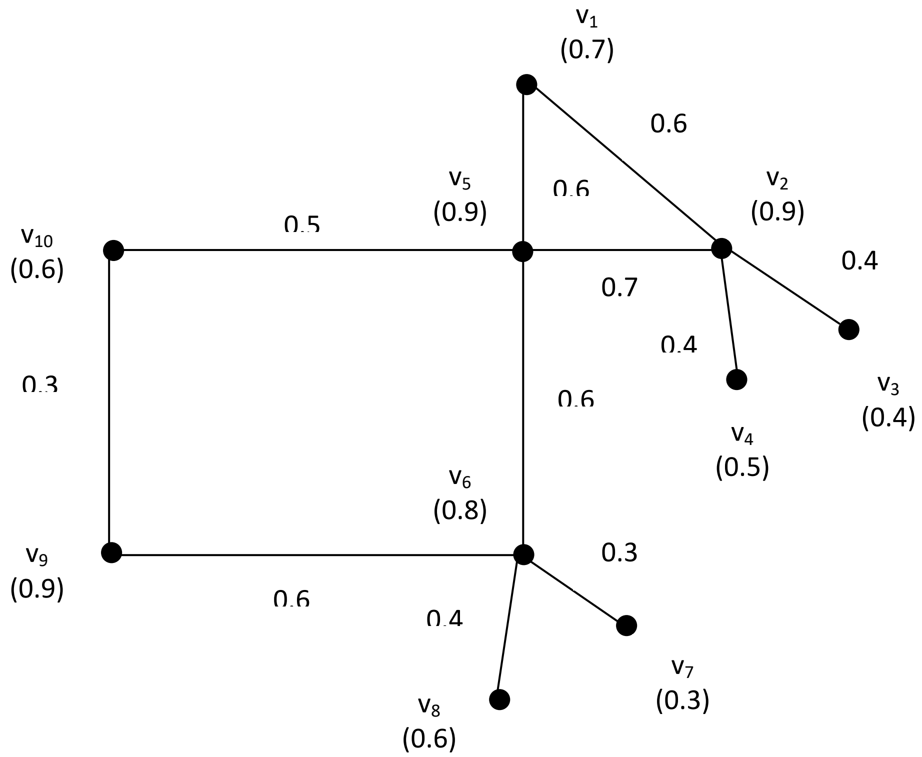

Figure (2)

Theorem 4.15 Let G be connected fuzzy graph with order p and p_1 be the order of end vertices then $\gamma_{cee}(G) \geq p_1 + \min \sigma(x)$, where $x \in V$ which is not end vertex.

Proof Clearly if G is connected fuzzy graph with order p have their following two cases :

Case (1) : If G has end vertices. Let D be any connected end equitable dominating set of G then p_1 be order of end vertices and also supporting vertices must along to dominating set. Hence $\gamma_{cee}(G) \geq p_1 + \min \sigma(x)$

Case (2) : If G has no end vertices then it is obviously true. i.e., $\gamma_{cee}(G) \geq p_1 + \min \sigma(x)$

Conclusion The concept of domination in fuzzy graph is very rich both in theoretical developments and applications. In this paper we introduce the concept connected end equitable domination in fuzzy graphs. We have found some bounds for the connected end domination number of fuzzy graphs. The various types of equitable domination in fuzzy graphs to precede future result.

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